Multiple Aligned Characteristic Curves for Surface Fairing

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Abstract. Characteristic curves like isophotes, reflection lines and reflection circles are well–established concepts which have been used for automatic fairing of both parametric and piecewise linear surfaces. However, the result of the fairing strongly depends on the choice of a particular family of characteristic curves: isophotes or reflection lines may look perfect for a certain orientation of viewing and projection direction, but still have imperfections for other directions. Therefore, fairing methods are necessary which consider multiple families of characteristic curves. To achieve this, we first introduce a new way of controlling characteristic curves directly on the surface. Based on this, we introduce a fairing scheme which incorporates several families of characteristic curves simultaneously. We confirm effectiveness of our method for a number of test data sets.

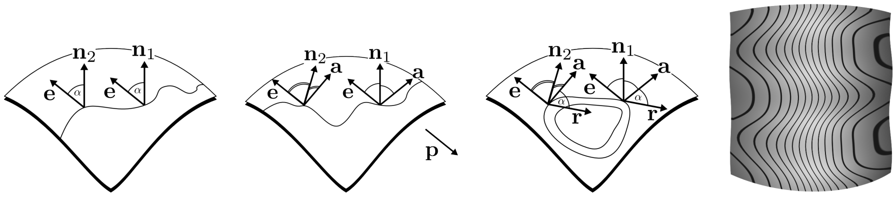
1 Introduction

Visualization of characteristic curves provides a valuable and important tool for first–order surface interrogation (see [1] for a recent survey). Inspection of characteristic surface curves allows for rating and improving surface design as well as for intuitive detection of surface defects: on the one side, they simulate aesthetic appearance under certain lighting conditions and environment, while on the other hand continuity and smoothness of these curves visualize respective differential properties for surface derivatives.

Characteristic surface curves like reflection lines were originally (and still are) used for interrogation and design of physical models, and the concept is simulated for CAGD models in a virtual environment. Surprisingly these curves are mainly used for interrogation, and only few approaches exist which apply them for surface fairing and design [2–4].

Yet, the proposed methods that take advantage of characteristic curves in this setting all have in common that they only consider a single curve family, i.e., a main direction represented by these curves. This results in an optimized behavior of the curves for this single direction, but — as we will show in this paper — the single direction fairing does in general not also yield an optimized characteristic of all other curve directions at the same optimized location. In fact, our experiments indicate that the reverse is true.

Section 4 presents a new fairing scheme for triangulated surfaces that is capable of incorporating an arbitrary number of families *simultaneously*. Prior to that, for efficient use of this scheme in practice, in section 3 we develop intuitive methods for real–time curve control, i.e., determining parameters such that specific interpolation or alignment constraints on the surface are fulfilled. These methods allow for interactive and automatic curve specification.



(a) Isophotes (b) Reflection lines (c) Reflection circles (d) Isophotes example

Fig. 1: Definitions of characteristic curves and example for family of isophotes on a wavy cylinder

1.1 Related Work

In this paper, we consider isophotes, reflection lines, and reflection circles. All these classes of characteristic curves are illumination curves since every curve originates from light–surface interaction [5].

Isophotes can be regarded as surface curves of constant incident light intensity which were extensively used to detect surface imperfections [6, 5, 1].

The reflection of a straight line on a surface is called reflection line. Just as isophotes, reflection lines possess special properties making them valuable for surface interrogation and surface fairing applications of parametric [2, 3, 7] and piecewise linear surfaces[4]. Recently, [4] applied reflection lines for fairing triangular meshes employing a screen–space surface parametrization. This work provides profound analysis of the arising numerical minimization and careful discretization of the emerging differential operators [8]. It is most similar yet different to this work.

Reflection circles arise from the reflections of concentric circles on a surface similar to reflection lines. Although reflection circles are the more general class of surface curves [9], they haven't been used as thorough as the other more specialized classes in surface–fairing applications. Still, recently [10] argue that a simplified version of reflection circles called circular highlight lines also performs well in surface–fairing applications.

There is vast literature on general surface denoising and fairing methods as well as fair surface design based on polygonal meshes, which we do not consider here but instead refer to a recent survey [11]. Similarly, we do not discuss alternative use of light lines such as surface reconstruction applications (see, e.g., [12]).

2 Characteristic Curves

We use definitions of characteristic curves — isophotes, reflection lines, and reflection circles (see figure 1) — which only depend on the normal directions of the surface and not on its position [9]. This means we assume that both, viewer and light sources (which are lines and circles), are located at infinity. This is a common simplification for various kinds of environment mapping. In the following, $\mathbf{e}$ denotes the normalized eye vector (viewing direction), and $\mathbf{n}(u, v)$ is the unit normal to the surface $\mathbf{x}(u, v)$.

Isophotes are surface curves of constant incident light intensity essentially taking into account Lambert's cosine law or diffuse lighting. Given are eye direction $\mathbf{e}$ and an angle α, then an isophote consists of all surface points $\mathbf{x}(u, v)$ satisfying $\mathbf{e} \cdot \mathbf{n}(u, v) = \cos \alpha$. Variation of angle α yields a *family of isophotes*.

Reflection lines are surface curves showing the mirror image of a line shaped light source. Given are eye direction $\mathbf{e}$ and a line at infinity defined by its unit normal $\mathbf{p}$, then a reflection line consists of all surface points satisfying $\mathbf{a} \cdot \mathbf{p} = 0$ with $\mathbf{a} = 2\,(\mathbf{e} \cdot \mathbf{n})\ \mathbf{n} - \mathbf{e}$. Variation of $\mathbf{p}$ along a line at infinity yields a *family of reflection lines*.

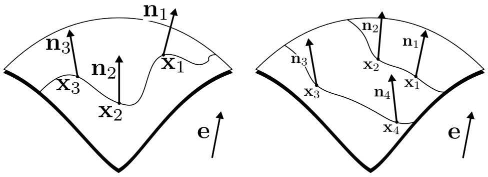
Reflection circles [9] provide a generalization of isophotes and reflection lines. They can be considered as mirror images of a family of concentric circles on the surface. Given are $\mathbf{e}$ and a circle at infinity defined by a normalized center direction $\mathbf{r}$ and an angle α, then a reflection circle consists of all surface points satisfying $\mathbf{a} \cdot \mathbf{r} = \cos \alpha$ with $\mathbf{a} = 2\,(\mathbf{e} \cdot \mathbf{n})\ \mathbf{n} - \mathbf{e}$. This can be easily transformed to the condition $(\mathbf{e} \cdot \mathbf{n})\ (\mathbf{r} \cdot \mathbf{n}) = v$, where $v = \frac{1}{2}(\cos \alpha + \mathbf{e} \cdot \mathbf{r})$. Reflection circles provide generalizations of other classes of characteristic curves in a sense for $\mathbf{r} = \mathbf{e}$ or $\mathbf{r} = -\mathbf{e}$ they are equivalent to isophotes, whereas for $\mathbf{r} \cdot \mathbf{e} = 2v$ they are equivalent to reflection lines, respectively. *Families of reflection circles* are obtained by either variation of v within range $[-1, 1]$, or variation of $\mathbf{a}$, or simultaneous variation of both parameters, respectively. In the following we will consider only the first option of varying the scalar parameter v.

3 Characteristic Curve Control

For virtual surface interrogation, e.g., using reflection lines, a simple environment map is sufficient to show families of reflection lines while the user moves the geometric object under inspection. This is simple and intuitive. However, in our setting of surface fairing, we require specification of certain characteristic curves: for a region in focus the user wants to specify curves quickly and intuitively such that they are roughly aligned with a prescribed direction. This setting provides a problem of its own because the defining parameters of the curves do not directly relate to the resulting pathway on the surface. Moreover plain parameter variation often yields counterintuitive and unexpected results.

In this section we show how to facilitate control of characteristic curves on surfaces in order to enable their intuitive use in practice.

The basic idea of every presented alignment method is to let a user or a (semi-)automatic operation specify a small number of points on the surface which a curve or family of curves shall pass through. Such points will be called *selections*. Then parameters of the curves are calculated by different alignment methods from surface normals in a way that the respective defining conditions are satisfied. Alignment methods differ in the number of required point selections and in their semantics in relation to the curve class. As every alignment method only depends on a small, constant number of points and specifically on the surface normals in these points, they are independent of the complexity of the surface the controlled curves are embedded in. In practice, the user selects by ray intersections with the surface, and selections can be dragged on the surface to fine–tune a curve alignment in real–time. In a similar way, a stencil of selection prototypes can be projected onto the surface for automatic curve control.



(a) $(ISOP, 3x)$ (b) $(ISOP, 2x + 2x)$

Fig. 2: Isophotes alignment methods

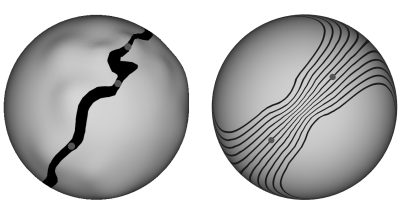


Fig. 3: Alignment examples using $(ISOP, 3x)$ and $(RECI, 2x$ + constr)

3.1 Alignment of Isophotes

It turns out that three selections on the surface are sufficient to define a general isophote passing through these points. We give a closed form expression which yields the parameter $\mathbf{e}$ and $\cos\alpha$. Let $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ be three selections on a smooth surface and $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ the respective unit surface normals. Then

$$\begin{aligned} \mathbf{e} &= \frac{(\mathbf{n}_1 \times \mathbf{n}_2) + (\mathbf{n}_2 \times \mathbf{n}_3) + (\mathbf{n}_3 \times \mathbf{n}_1)}{\|(\mathbf{n}_1 \times \mathbf{n}_2) + (\mathbf{n}_2 \times \mathbf{n}_3) + (\mathbf{n}_3 \times \mathbf{n}_1)\|} \quad \text{and} \\ \cos\alpha &= \mathbf{e}\cdot\mathbf{n}_1 = \mathbf{e}\cdot\mathbf{n}_2 = \mathbf{e}\cdot\mathbf{n}_3 \end{aligned} \tag{1}$$

are the parameters defining an isophote $(\mathbf{e}, \cos\alpha)$ interpolating $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$.

In the remainder we refer to this alignment method as $(ISOP, 3x)$, indicating that an isophote is aligned using three selections. Figure 2a illustrates the configuration.

The alignment of a family of isophotes is achieved using two pairs of selections: Let $(\mathbf{x}_1, \mathbf{x}_2)$ and $(\mathbf{x}_3, \mathbf{x}_4)$ be two pairs of selections on a smooth surface and $(\mathbf{n}_1, \mathbf{n}_2)$ and $(\mathbf{n}_3, \mathbf{n}_4)$ the respective unit surface normals. Then

$$\begin{aligned} \mathbf{e} &= \frac{(\mathbf{n}_1 - \mathbf{n}_2) \times (\mathbf{n}_3 - \mathbf{n}_4)}{\|(\mathbf{n}_1 - \mathbf{n}_2) \times (\mathbf{n}_3 - \mathbf{n}_4)\|} \quad , \\ \cos\alpha_1 &= \mathbf{e}\cdot\mathbf{n}_1 = \mathbf{e}\cdot\mathbf{n}_2 \quad \text{and} \\ \cos\alpha_2 &= \mathbf{e}\cdot\mathbf{n}_3 = \mathbf{e}\cdot\mathbf{n}_4 \end{aligned} \tag{2}$$

are the parameters defining two isophotes $(\mathbf{e}, \cos\alpha_1)$ and $(\mathbf{e}, \cos\alpha_2)$ of the same family passing through the points $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$, $\mathbf{x}_4$, respectively. We call this method $(ISOP, 2x + 2x)$ because two isophotes of the same family are aligned requiring two selections for each curve. Two isophotes of the same family aligned using this method are depicted in figure 2b.

3.2 Constraint Alignment of Reflection Lines and Circles

Due to their relative simplicity, isophotes constitute a special case for which closed form solutions to the general alignment can be given. In contrast, general alignment of reflection lines and circles requires root finding of higher order polynomials to determine parameters. Hence, no general closed form expressions can be given. Instead, we present a constraint approach.

The constraint approach requires only two selections in order to align both, either reflection lines or reflection circles on smooth surfaces. Reflection circles are a generalization of reflection lines: setting its $\cos\alpha$ parameter to zero in fact specifies a reflection line with one lost degree of freedom which can be taken advantage of afterwards. We restrict the derivation of the alignment method to reflection lines in the first place and make it applicable for both curve classes by variation of the extra parameter. Let $\mathbf{x}_1$, $\mathbf{x}_2$ be two selections on a smooth surface and $\mathbf{n}_1$, $\mathbf{n}_2$ their linear independent normals. Then

$$\mathbf{e} = \frac{(\mathbf{n}_1 + \mathbf{n}_2)}{\|(\mathbf{n}_1 + \mathbf{n}_2)\|} \quad \text{and} \quad \mathbf{r} = \mathbf{p} = \frac{(\mathbf{n}_1 \times \mathbf{n}_2)}{\|(\mathbf{n}_1 \times \mathbf{n}_2)\|} \tag{3}$$

are the parameters defining a reflection line $(\mathbf{e}, \mathbf{p})$ as well as a reflection circle $(\mathbf{e}, \mathbf{r}$, $\cos\alpha = 0)$ passing through the points $\mathbf{x}_1$, $\mathbf{x}_2$. These alignment methods are referred to as $(REFL, 2x+$ constr) and $(RECI, 2x+$ constr), respectively. We call the approach constraint as the parameter vectors of eye vector $\mathbf{e}$ and normal $\mathbf{p}$ of the line at infinity are restricted to be perpendicular, so $\mathbf{e} \cdot \mathbf{p} = 0$. Geometrically this means that the eye point at infinity is constraint to the respective line at infinity.

4 Surface Fairing

We define the goal of our surface fairing method as follows: a smooth surface should be altered by minimal local displacements such that pathways of characteristic curves are straightened and homogenized. Therefore, our aim is to penalize curvature of characteristic curves.

Let $\mathcal{C}$ define a set of discrete families of curves f, e.g., specified by a finite set of angles. We consider piecewise linear surfaces $\mathcal{M}$, i.e., triangles meshes defined by $(\mathcal{V}, \mathcal{E}, \mathcal{F})$, sets of vertices, oriented edges, and faces, respectively. Then we define discrete error functionals as

$$E(\mathcal{V}) = \sum_{f \in \mathcal{C}} E_f(\mathcal{V}) \quad \text{with} \quad E_f(\mathcal{V}) = \sum_{v \in \mathcal{V}} \kappa_f^2\,(\mathbf{v}) \ , \tag{4}$$

where $\mathbf{v}$ denotes the position of vertex $v \in \mathcal{V}$ and $\kappa_f\,(\mathbf{v})$ is the curve curvature of the family member of f at $\mathbf{v}$. We call E_f *family error* and E *accumulated error*, respectively. Minimizing $E(\mathcal{V})$ by altering vertex positions yields an optimized, fair surface $\mathcal{M}'$.

Curvature of Characteristic Curves

Each family of characteristic curves defines a piecewise linear scalar field over the surface, i.e., the defining equations are evaluated at every vertex. Then members of the family are given implicitly as iso-curves w.r.t. to a certain isovalue. We approximate curvature of such characteristic iso-curves per vertex $v \in \mathcal{V}$ as follows.

We find intersections of the iso-contour $\mathbf{c}(\mathbf{v}) = c_v$ with the edges $(i, j) \in \mathcal{E}_v^1$ bounding the 1–ring neighborhood $\mathcal{N}_v$ of v by linear interpolation between $\mathbf{c}(v_i) = c_{v_i}$ and $\mathbf{c}(v_j) = c_{v_j}$. From $\mathbf{v}$ and the positions of two intersections, curvature $\kappa_f(\mathbf{v})$ is given as the inverse radius of the interpolating circle. If the intersections are approximately collinear, i.e., circle degenerates to a line, we assume zero curvature.

Local Optimization by Vertex Displacement

In order to minimize the accumulated error E, we iteratively displace vertices along their normal direction. We analyze the local setting for vertex v and its neighborhood. The surface normal $\mathbf{n}(\mathbf{v}) := \mathbf{n}_v$ is approximated as the average of weighted triangle normals, i.e., $\tilde{\mathbf{n}}_v = \sum_{(i,j)\in\mathcal{E}_{\mathbf{v}}^1} (\mathbf{v}_i - \mathbf{v}) \times (\mathbf{v}_j - \mathbf{v})$, and $\mathbf{n}_v = \frac{\tilde{\mathbf{n}}_v}{||\tilde{\mathbf{n}}_v||}$. Note that $\mathcal{E}_v^1$ includes all directed (counter-clockwise oriented) edges bounding the 1–ring of v. For simplicity we use an area weighting scheme here, however, applying more sophisticated normal approximation methods (see, e.g., survey [1]) yields similar formulas.

Displacing vertices as $\mathbf{v}' = \mathbf{v} + \epsilon\mathbf{n}_v$ for small scalar ϵ entails recomputation of vertex normals only within the 1–ring of v. It is easy to see that curvatures, however, are effected within the 2–neighborhood $\mathcal{N}_v^2$ and curvature variation is therefore locally bounded. Consequently, scalar values within the 3–neighborhood $\mathcal{N}_v^3$ of v have to be considered for computing the global variation of the error induced by the displacement.

We derive the following expression

$$\tilde{\mathbf{n}}_v\left(\epsilon\right) = \sum_{(i,j)\in\mathcal{E}_v^1} \mathbf{v}_i \times \mathbf{v}_j + \epsilon\left(\delta_{ik}\left(\mathbf{n}_{v_k} \times \mathbf{v}_j\right) - \delta_{jk}\left(\mathbf{n}_{v_k} \times v_i\right)\right) \tag{5}$$

as updated unnormalized normal direction of vertex v after displacement of vertex v_k by $\epsilon\,\mathbf{n}_{v_k}$. The normal of the displaced vertex remains constant.

For fairing, a vertex is iteratively translated in several ϵ-steps as long as a single displacement reduces the global error.

Mesh Fairing

We use the analysis of the local setting to globally minimize the accumulated error $E(\mathcal{V})$ for all vertices (or for those within a region of interest, respectively). We take a randomized and serialized approach which iterates the following steps:

1. Randomly pick a vertex $v \in \mathcal{V}$.
2. Take a binary decision whether a translation direction of $\mathbf{v}$ in direction $\mathbf{n}_v$ or $-\mathbf{n}_v$ makes E decrease; otherwise restart at step 1.
3. Find the most effective displacement of $\mathbf{v}$ by integrating ϵ-steps as long as the global error reduction is of significant magnitude. The step size ϵ is adapted during integration by logarithmical attenuation depending on the error reduction rate.

We terminate the global iteration if no more enhancement can be achieved over a specific number of iterations. Our experiments show that $E(\mathcal{V})$ is effectively reduced at reasonable computational cost (see section 5). We remark that in every local optimization step curvature of characteristic curves is reduced not only for vertex v but also in its 2–neighborhood due to the overlap of the respective curvature stencils. Hence, for optimization within a region of interest, the boundary region is automatically processed such that smooth transition of optimized curves across the boundary is ensured.

5 Results

Curve control. The proposed alignment methods highly facilitate the interrogation of surfaces by characteristic surface curves. Figure 3 gives two examples (see also supplemental video). We found $(ISOP, 3x)$ and $(RECI, 2x + \text{constr})$ especially useful for automatic surface fairing applications as they require a low number of selections, are computable in constant real-time and yield stable alignment results.

Single family fairing. To begin with, a single family fairing of a Chevrolet Corvette C4[1] engine hood by reflection circles, which were aligned by $(RECI, 2x{+}\text{constr})$, is shown in figure 4. Within 2000 iterations the accumulated error dropped by 75.91% for about 500 optimized vertices; the processing time was $40s$. All timings were measured on a 2.2GHz AMD Opteron processor.

Multiple family fairing. One of our main goals is to show that *multiple families* of characteristic curves should be considered simultaneously. So far, only a single family had been used in prior work. We demonstrate that the latter generally yields improvement of this single family only, while other families may improve or not — or may even get worse in appearance. Figure 5 shows an example, where three differently aligned families (using $(ISOP, 3x)$) of isophotes are shown on the initial model of a BMW Z3 engine hood in the top row. The second row shows families resulting from solely fairing the left family: the two other families did not improve in the same way. This is because fairing of a surface by a single curve family does not necessarily improve the overall reflective properties of a surface. A subsequent example shows that the contrary can also be the case. Incorporating all three families into the fairing process gives better overall results, see bottom row. The total processing time for multiple families depends linearly on their number. The accumulated error of all families dropped by 32.5% after fairing the single family using 2000 iterations, however, it dropped by 63.7% fairing all directions. With the model scaled to the unit sphere, the average displacement per vertex is of length $8.33 \cdot 10^{-6}$, hence the induced approximation error to the initial surface is negligible.

In figure 6 we analyze the fairing of the car roof of a Volkswagen Beetle using multiple families of reflection circles which were automatically aligned by $(RECI, 2x + \text{constr})$ to be uniformly distributed. This example illustrates several different families, and it illustrates several facts: first, the benefits of simultaneous optimization of multiple families, second, the potential corruption of families if only a single other family is considered. In the example the vertical family shows already good quality which degrades when only the horizontal family is optimized. In addition, the behavior of two skew families is shown, in the final experiment they are also considered in optimization. The error is plotted versus the number of iterations for all settings, accumulated

[1] Corvette and BMW models from www.dmi3d.com.

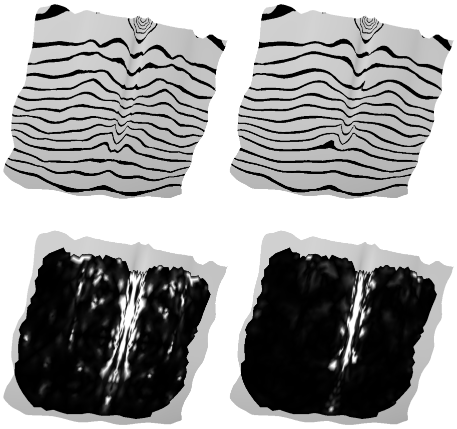


Fig. 4: Reflection circles on a Chevrolet Corvette C4 engine hood before (left) and after (right column) fairing

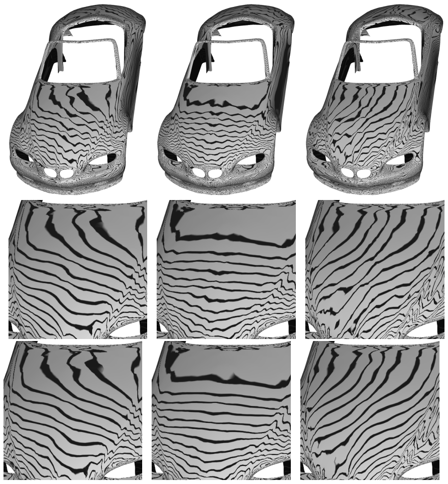


Fig. 5: Fairing of BMW Z3 model hood: Initial three families (first row), only first family faired (second row) and all families faired (third row)

error dropped by 32.26%, 41.32% and 58.28%, respectively. We moreover found that no other intermediate family direction showed an imperfect behavior on the surface faired this way.

Curve class comparison. Both, isophotes and reflection circles, can be faired by our generic approach. In our experiments cross validation showed comparable performance for both curve classes. We could not affirm the proposition in [10] stating that circular highlight lines are better suited for surface fairing than highlight lines, as all directions are captured.

6 Discussion.

Our results support our claim that simultaneous consideration of multiple curve families is advantageous for surface fairing. Furthermore, we provide new methods for controlling characteristic curves, which haven't been applied in any previous approach.

Prior work most similar to our method is [4] who considered reflection lines for shape optimization based on triangle meshes. They concentrate on optimizing one single family of reflection lines. The family is provided by the user, control of curve parameters is not discussed. Emphasis on discretization and efficient numerical minimization using screen–space parametrization and other approximations yields a real–time algorithm with some view dependent limitations. In contrast our focus was on new aspects summarized above. Our optimization method uses a far simpler randomized greedy surface optimization which converges to local minima and is far from real–time application. It would be an interesting project for future work to see whether our method could be combined with the minimization framework in [4].

7 Conclusions

In this paper we make the following contributions:

- We showed that the fairing of a particular family of characteristic surface curves (like isophotes, reflection lines, or reflection circles) does not necessarily yield a fairer surface in the sense that other families of surface curves become fairer as well.
- We introduced a number of techniques to align characteristic curves on surfaces by directly placing and interactively moving points on the surface instead of specifying viewing and projection parameters.
- Based on this, we presented an approach for simultaneous fairing of multiple families of characteristic surface curves which gives better results than a single-family fairing.

The following issues remain open for future research:

- Although the whole fairing process can be considered as a preprocess which is carried out once, the performance of the algorithm could be enhanced.
- We have no general solution on the question how many families should be faired simultaneously to get optimal results. Clearly, increasing the number of families enhances the results but also increases the computing time linearly. In all our examples, four families were sufficient to ensure the fairness of all families. However, we do not have a theoretical confirmation of this statement yet.

References

1. Hahmann, S., Belyaev, A., Buse, L., Elber, G., Mourrain, B., Rössl, C.: 1. Mathematics and Visualization. In: Shape Interrogation. Springer, Berlin, Germany (2008) 1–52
2. Klass, R.: Correction of local surface irregularities using reflection lines. Computer-Aided Design **12** (1980) 73–77
3. Kaufmann, E., Klass, R.: Smoothing surfaces using reflection lines for families of splines. CAGD **20** (1988) 312–316
4. Tosun, E., Gingold, Y., Reisman, J., Zorin, D.: Shape optimization using reflection lines. In: Symposium on Geometry Processing. (2007) 193–202
5. Hagen, H., Schreiber, T., Gschwind, E.: Methods for surface interrogation. IEEE Visualization (1990) 187–193
6. Poeschl, T.: Detecting surface irregularities using isophotes. CAGD **1** (1984) 163–168
7. Theisel, H., Farin, G.: The curvature of characteristic curves on surfaces. IEEE Computer Graphics and Applications **17** (1997) 88–96
8. Grinspun, E., Gingold, Y., Reisman, J., Zorin, D.: Computing discrete shape operators on general meshes. Eurographics **25** (2006) 547–556
9. Theisel, H.: Are isophotes and reflection lines the same? Comput. Aided Geom. Des. **18** (2001) 711–722
10. Nishiyama, Y., Nishimura, Y., Sasaki, T., Maekawa, T.: Surface faring using circular highlight lines. Computer-Aided Design and Application **7** (2007) 405–414
11. Botsch, M., Pauly, M., Kobbelt, L., Alliez, P., Lévy, B., Bischoff, S., Rössl, C.: Geometric modeling based on polygonal meshes. In: SIGGRAPH Course Notes. (2007)
12. Halstead, M.A., Barsky, B.A., Klein, S.A., Mandell, R.B.: Reconstructing curved surfaces from specular reflection patterns using spline surface fitting of normals. In: SIGGRAPH. (1996) 335–342

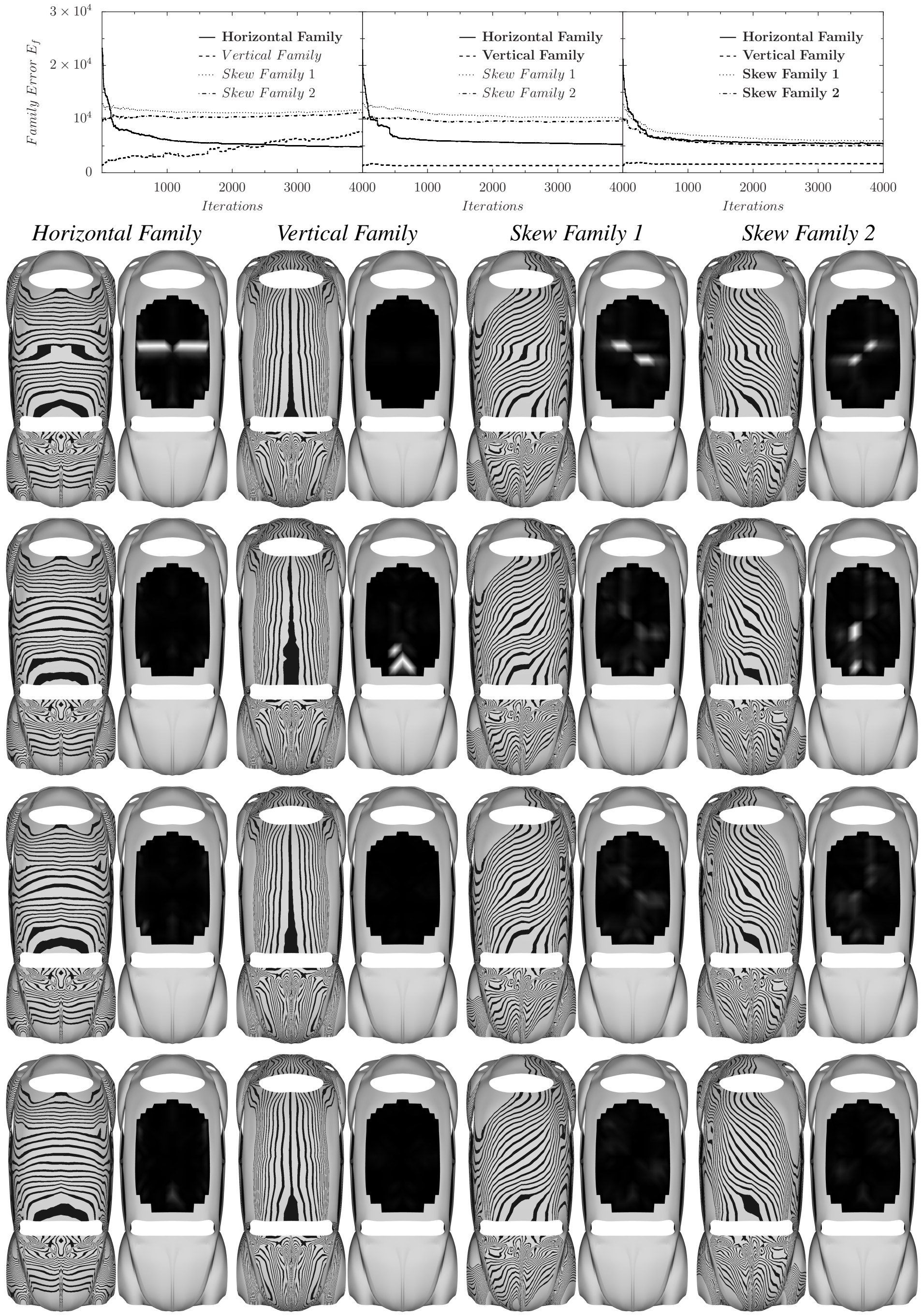


Fig. 6: Optimization of the Volkswagen Beetle car roof. Bold family names in the error plot correspond to optimized families. The top picture row shows initial curve families with a plot of their respective curve curvatures. Solely fairing of the horizontally oriented family corrupts the vertically aligned family (left plot, second row). Simultaneous fairing of both families enhances appearance the horizontal family while preserving the quality of the vertical family (center plot, third row). Even better results can be achieved by considering also the two skew families (right plot, last row)