A.4 Proof of Equations (56)–(59)

The theoretical core of our approach are the closed-form solutions (56)–(59) and (70) of the integrals (50)–(55). The proofs can be obtained by direct a application of (74)–(76). To exemplify this, we show the equivalence of (50) and (56) by recursively solving the integrals in (50) from the most inner to the most outer one. We rewrite (50) as

$$\mathbf{m} = \int_{\mathrm{I\!R}^n} \cdots \int_{\mathrm{I\!R}^n} h(\mathbf{x}_2, \ldots, \mathbf{x}_N)\, d\mathbf{x}_2 \,\ldots\, d\mathbf{x}_N \tag{77}$$

with

$$h(\mathbf{x}_2,...,\mathbf{x}_N) = \int_{\mathrm{I\!R}^n} (p_1(\mathbf{x}_1)\cdot \ldots \cdot p_N(\mathbf{x}_N)\cdot \widetilde{\mathbf{m}})\; d\mathbf{x}_1 \tag{78}$$

$$= \int_{\mathrm{I\!R}^n} \left(p_1(\mathbf{x}_1)\cdot \ldots \cdot p_N(\mathbf{x}_N)\cdot \frac{\mathbf{x}_1 + \ldots + \mathbf{x}_N}{N} \right) d\mathbf{x}_1$$

$$= \int_{\mathrm{I\!R}^n} p_1(\mathbf{x}_1)\cdot \mathbf{r}(\mathbf{x}_1)\; d\mathbf{x}_1 \tag{79}$$

where

$$\mathbf{r}(\mathbf{x}_1) = p_2(\mathbf{x}_2)\cdot \ldots \cdot p_N(\mathbf{x}_N)\, \frac{\mathbf{m}_1 + \mathbf{x}_2 + \ldots + \mathbf{x}_N}{N} + p_2(\mathbf{x}_2)\cdot \ldots \cdot p_N(\mathbf{x}_N)\, \frac{\mathbf{x}_1 - \mathbf{m}_1}{N}. \tag{80}$$

This shows that $\mathbf{r}(\mathbf{x}_1)$ is linear in $\mathbf{x}_1$. Applying (74)–(76) gives that only the constant coefficient of $\mathbf{r}(\mathbf{x}_1)$ is relevant for h, yielding the following closed form for (79):

$$h(\mathbf{x}_2, \ldots, \mathbf{x}_N) = p_2(\mathbf{x}_2 \cdot \ldots \cdot p_N(\mathbf{x}_N)\, \frac{\mathbf{m}_1 + \mathbf{x}_2 + ... + \mathbf{x}_N}{N}. \tag{81}$$

With this, the most inner integral of (50) is resolved. By recursively repeating the procedure to the next inner integral, we can finally resolve all integrals and end with

$$\mathbf{m} = \frac{\mathbf{m}_1 + \ldots + \mathbf{m}_N}{N} = \overline{\mathbf{m}} \tag{82}$$

which proves (56). The proof of (57)–(59) and (70) follows the same idea of applying (74)–(76) to recursively resolve the unbounded integrals from the most inner one to the most outer one. Since the computations are lengthy but straightforward, we proved a Maple sheet `maple02.txt` in the additional material. Note that this sheet also encodes the proofs the closed form solutions in existing uncertainty-aware PCA, in particular (16), and (17).

A.5 Performance Measures

Average runtime in seconds for sampling the distributions shown in Figure 10, using an AMD Epyc 7543:

	$\mathbf{R = 0.2}$	$\mathbf{R = 0.3}$	$\mathbf{R = 0.5}$	$\mathbf{R = 1.0}$
$\mathbf{U = 10^3}$	1.56	1.54	1.52	1.50
$\mathbf{U = 10^4}$	15.44	15.34	15.49	15.28
$\mathbf{U = 10^5}$	154.99	154.87	152.55	150.07
$\mathbf{U = 10^6}$	1548.87	1542.81	1526.83	1500.29